Theoretical approach to microwave radiation-induced zero-resistance states in 2D electron systems

J. Iñarrea[1,2] and G. Platero[1]
[1]Instituto de Ciencia de Materiales, CSIC, Cantoblanco, Madrid, 28049, Spain
[2]Escuela Politécnica Superior, Universidad Carlos III, Leganes, Madrid.
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We present a theoretical model in which the existence of radiation-induced zero-resistance states is analyzed. An exact solution for the harmonic oscillator wave function in the presence of radiation, and a perturbation treatment for elastic scattering due to randomly distributed charged impurities, form the foundations of our model. Following this model most experimental results are reproduced, including the formation of resistivity oscillations, their dependence on the intensity and frequency of the radiation, temperature effects, and the locations of the resistivity minima. The existence of zero-resistance states is thus explained in terms of the interplay of the electron MW-driven orbit dynamics and the Pauli exclusion principle.

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In the last two decades, especially since the discovery of the Quantum Hall Effect, a lot of progress has been made in the study of two-dimensional electron systems (2DES), and very important and unusual properties have been discovered when these systems are subjected to external AC and DC fields. In the last two years two experimental groups[1,2] have announced the existence of vanishing resistance in 2DES, i.e. zero resistance states (ZRS), when these systems are under the influence of a moderate magnetic field (B) and microwave (MW) radiation simultaneously. In the same kind of experiments large resistivity oscillations have been observed[1,2,3,4].
The discovery of this novel effect has led to a great deal of theoretical activity, and among the most interesting contributions we can summarize various different approaches. Some, like Girvin et al,[5,6] argue that this striking effect has to do with photon-assisted scattering from impurities or disorder, or alternatively arises from acoustic phonon scattering[7]. Others[8,9] relate the ZRS with a new structure of the density of states of the system in the presence of light. According to Andreev's approach[10], the key is the existence of an inhomogeneous current flowing through the sample due to the presence of a domain structure in it. To date there is no consensus about the true origin. All the theories above have in common that they predict negative resistivity, while this has not been experimentally confirmed. Only Willett et al[11], have recently observed negative conductivity for certain configurations of contacts.
In this Letter we develop a semi-classical model that is based on the exact solution of the electronic wave function in the presence of a static B interacting with MW-radiation, i.e. a quantum forced harmonic oscillator, and a perturbation treatment for elastic scattering from randomly distributed charged impurities. We explain and reproduce most experimental features, and clarify the physical origin of ZRS. For large MW field amplitudes, final states for electrons, which semi-classically describe orbits whose center positions oscillate due to the MW field, will be occupied. The MW field thus blocks the electron movement between orbits, and the longitudinal conductivity and resistivity ρ_{xx} will be zero (see Fig.1). We explain the dependence of ρ_{xx} on temperature (T) by means of the electron interaction with acoustic phonons which acts as a damping factor for the forced quantum oscillators, and we discuss the ZRS dependence on B. We observe them at $w/w_c = j + 1/4$ for the experimental parameters of ref.[1], being w the MW frequency, w_c the cyclotron frequency and j an integer. In our model we do not consider the induced electrostatic potentials by the charge distribution within the sample, nor the dynamical electronic response induced by the AC-potential.[12,13]
We first obtain an exact expression of the electronic wave vector for a 2DES in a perpendicular B, a DC electric field and MW radiation which is considered semi-classically. The total hamiltonian H can be written as:

$$
\begin{aligned}
H &= \frac{P_x^2}{2m^*} + \frac{1}{2}m^* w_c^2 (x-X)^2 - eE_{dc}X + \\
&+ \frac{1}{2}m^* \frac{E_{dc}^2}{B^2} - eE_0 \cos wt (x-X) - \\
&- eE_0 \cos wt X \\
&= H_1 - eE_0 \cos wt X
\end{aligned}
\tag{1}
$$

X is the center of the orbit for the electron spiral motion: $X = \frac{\hbar k_y}{eB} - \frac{eE_{dc}}{m^* w_c^2}$, E_0 the intensity for the MW field and E_{dc} is the DC electric field in the x direction. H_1 is the hamiltonian corresponding to a forced harmonic oscillator whose orbit is centered at X. H_1 can be solved exactly[14,15], and using this result allows an exact solution for the electronic wave function of H to be obtained:

$$
\begin{aligned}
&\Psi(x,t) = \phi_n(x - X - x_{cl}(t), t) \\
&\times exp\left[i\frac{m^*}{\hbar}\frac{dx_{cl}(t)}{dt}[x - x_{cl}(t)] + \frac{i}{\hbar}\int_0^t L dt' \right] \\
&\times \sum_{m=-\infty}^{\infty} J_m \left[\frac{eE_0}{\hbar} X \left(\frac{1}{w} + \frac{w}{\sqrt{(w_c^2 - w^2)^2 + \gamma^4}} \right) \right] e^{imwt}
\end{aligned}
\tag{2}
$$

where γ is a phenomenologically-introduced damping factor for the electronic interaction with acoustic phonons,

ϕ_n is the solution for the Schrödinger equation of the unforced quantum harmonic oscillator and $x_{cl}(t)$ is the classical solution of a forced harmonic oscillator[15], $x_{cl} = \frac{eE_o}{m^* \sqrt{(w_c^2-w^2)^2+\gamma^4}} \cos wt$. L is the classical lagrangian, and J_m are Bessel functions. Apart from phase factors, the wave function for H is the same as the standard harmonic oscillator where the center is displaced by $x_{cl}(t)$. Now we introduce the impurity scattering suffered by the electrons in our model[16]. If the scattering is weak we can apply time dependent first order perturbation theory, starting from H as an exact hamiltonian and $\Psi_l(x,t)$ as the wave-vector basis. The aim is to calculate the transition rate from an initial state $\Psi_n(x,t)$, to a final state $\Psi_m(x,t)$:

$$W_{n,m} = \lim_{\alpha \to 0} \frac{d}{dt} \left| \frac{1}{i\hbar} \int_{-\infty}^{t'} < \Psi_m(x,t)|V_s|\Psi_n(x,t) > e^{\alpha t} dt \right|^2 \tag{3}$$

where V_s is the scattering potential for charged impurities[17]: $V_s = \sum_q \frac{e^2}{2S\epsilon(q+q_s)} \cdot e^{i\overrightarrow{q}\cdot\overrightarrow{r}}$, S being the surface of the sample, ϵ the dielectric constant and q_s is the Thomas-Fermi screening constant[17]. After some lengthy algebra we arrive at the following expression for the transition rate:

$$\begin{aligned} W_{n,m} &= \frac{e^5 n_i BS}{16\pi^2\hbar^2\epsilon^2} \left[\frac{\Gamma}{[\hbar w_c(n-m)]^2 + \Gamma^2} \right] \\ &\times \int_0^{q_{max}} dq \frac{q}{(q^2+q_0^2)^2} \frac{n_1!}{n_2!} e^{-\frac{1}{2}q^2R^2} \left(\frac{1}{2}q^2R^2 \right)^{n_1-n_2} \\ &\times \left[L_{n_2}^{n_1-n_2}(\frac{1}{2}q^2R^2) \right]^2 J_0^2(A_m) J_0^2(A_n) \end{aligned} \tag{4}$$

where $A_{n(m)} = \frac{eE_0}{\hbar} X_{n(m)} \left(\frac{1}{w} + \frac{w}{\sqrt{(w_c^2-w^2)^2+\gamma^4}} \right)$. With the experimental parameters we take, the arguments of the Bessel functions are very small ($\sim 10^{-2}$), and only J_0 terms need to be considered. Γ is the Landau level broadening, n_i is the impurity density and R is the magnetic characteristic length $R^2 = \frac{\hbar}{eB}$. $L_{n_2}^{n_1-n_2}$ are the associated Laguerre polynomials, $n_1 = max(n,m)$ and $n_2 = min(n,m)$.

Without radiation, an electron in an initial state Ψ_n corresponding to an orbit center position X_n^0, scatters and jumps to a final state Ψ_m with orbit center X_m^0, changing its average coordinate in the static electric field direction by $\Delta X^0 = X_m^0 - X_n^0 = q\cos\theta R^2$ (polar coordinates, q and θ, have been used). In the presence of MW radiation, the electronic orbit center coordinates change and are given according to our model by $X^{MW} = X^0 + x_{cl}(t)$. This means that due to the MW field all the electronic orbit centers in the sample oscillate back and forth in the x direction through x_{cl}. We have to consider two important factors: first, when an electron suffers a scattering process with a probability given by $W_{m,n}$, it takes a time $\tau = \frac{1}{W_{m,n}}$ for that electron to jump from an orbit center to another. Secondly, in the jump, as in

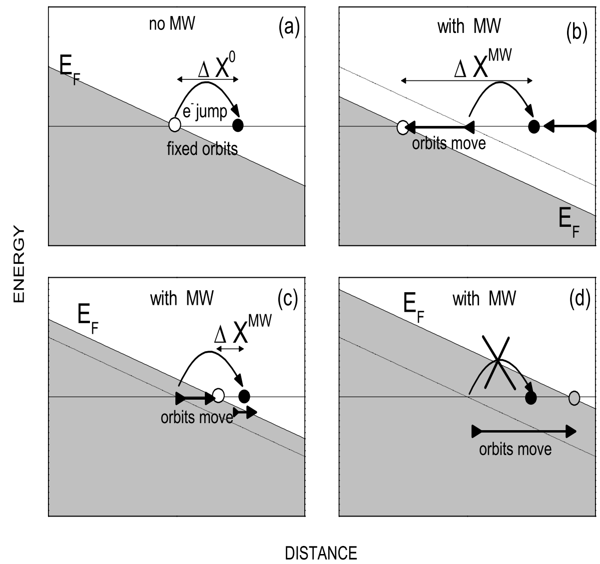

FIG. 1: Schematic diagrams of electronic transport without and with MW. In Fig.(1.a) no MW field is present, and due to scattering electrons jump between fixed-position orbits. When the MW field is on, the orbits are not fixed but oscillate at w. In Fig.(1.b) the orbits move backwards during the jump, and on average electrons advance further than in the no MW case. In Fig.(1.c) the orbits are moving forwards, and on average electrons advance less than in the no MW case. In Fig. (1.d) the orbits are moving forwards but their amplitudes are larger than the electronic jump, and the electron movement between orbits cannot take place because the final state is occupied. This situation corresponds to ZRS.

any other scattering event, the electron loses memory and phase with reference to the previous situation, and thus when it arrives at the final state the oscillation condition is going to be different from the starting point. If we consider that the oscillation is at its *mid-point* when the electron jumps from the initial state, and that it takes a time τ to get to the final one, then we can write for the average coordinate change in the x direction: $\Delta X^{MW} = \left(\Delta X^0 + \frac{eE_o}{m^*\sqrt{(w_c^2-w^2)^2+\gamma^4}} \cos w\tau \right)$.

In Fig. (1) we present schematic diagrams for the different situations. In Fig. (1.a) no MW field is present and electrons jump between fixed orbits, and on average an electron advances a distance ΔX^0. When MW field is on, the orbits are not fixed, and instead move back and forth through x_{cl}. Three cases can be distinguished. In Fig. (1.b) the orbits are moving backwards during the electron jump, and on average, due to scattering processes, the electron advances a larger distance than in the no MW case, $\Delta X^{MW} > \Delta X^0$. This corresponds to an increasing conductivity. In Fig. (1.c) the orbits are moving forwards and the electron advances a shorter dis-

tance, $\Delta X^{MW} < \Delta X^0$. This corresponds to a decrease in the conductivity with respect to the case without MW. If we increase the MW intensity, we will eventually reach the situation depicted in Fig. (1.d) where orbits are moving forwards, but their amplitude is larger than the electronic jump. In that case the electronic jump is blocked by the Pauli exclusion principle because the final state is occupied. This is the physical origin of ZRS. At different ranges of parameters, i.e. larger MW power or smaller w, additional terms corresponding to Bessel functions of order higher than zero would contribute to Eq. 4, which may eventually produce negative conductivity[11,18].
If the average value $\triangle X^{MW}$ is different from zero over all the scattering processes, the electron possesses an average drift velocity $v_{n,m}$ in the x direction[16]. This drift velocity can be calculated readily by introducing the term $\triangle X^{MW}$ into the integrand of the transition rate, and finally the longitudinal conductivity σ_{xx} can be written as: $\sigma_{xx} = \frac{2e}{E_{dc}} \int \rho(E_n) v_{n,m} [f(E_n) - f(E_m)] dE_n$. Gathering all the terms, we finally obtain the expression:

$$\sigma_{xx}(E_n) = \frac{e^7 n_i B^2 S}{16\pi^5 \epsilon^2 \hbar^3 E_{dc}} \sum_{n,m} \left[\frac{\Gamma}{[\hbar w_c (n-m)]^2 + \Gamma^2} \right]$$
$$\times \int dE_n \left[\frac{\Gamma}{[E_n - \hbar w_c (n + \frac{1}{2})]^2 + \Gamma^2} \right] [f(E_n) - f(E_m)]$$
$$\times \int_0^{q_m} dq \left[\frac{q(q\cos\theta R^2 + A\cos w\tau)}{(q+q_s)^2} \right]$$
$$\times \frac{n_1!}{n_2!} e^{-\frac{1}{2}q^2R^2} \left(\frac{1}{2} q^2 R^2 \right)^{n_1 - n_2} \left[L_{n_2}^{n_1-n_2} (\frac{1}{2} q^2 R^2) \right]^2$$
$$\times J_0^2(A_m) J_0^2(A_n) \qquad (5)$$

where the density of states $\rho(E_n)$ has been simulated by a Lorentzian function, being $A = \frac{eE_o}{m^* \sqrt{(w_c^2 - w^2)^2 + \gamma^4}}$. To obtain ρ_{xx} we use the relation $\rho_{xx} = \frac{\sigma_{xx}}{\sigma_{xx}^2 + \sigma_{xy}^2} \simeq \frac{\sigma_{xx}}{\sigma_{xy}^2}$, where $\sigma_{xy} \simeq \frac{n_i e}{B}$ and $\sigma_{xx} \ll \sigma_{xy}$. All our results have been based on experimental parameters corresponding to the experiments of Mani[1] et al. In Fig. 2 we show the magnetoresistivity ρ_{xx} obtained using our model, as a function of B for different MW field intensities, in all cases using the same frequency $w/2\pi = \nu = 103.5$ GHz. The darkness case is also presented. As the field intensity is lowered, the ρ_{xx} response decreases to eventually reach the darkness response. In the inset it is possible to see the calculated ρ_{xx} vs B^{-1}, which is roughly periodic in B^{-1} in agreement with experiment. The minima positions as a function of B are indicated with arrows, corresponding to $\frac{w}{w_c} = j + \frac{1}{4}$. In the minima corresponding to $j = 1$, ZRS are found. Although the qualitative behavior of ρ_{xx} as a function of B is very similar to the experimental one, the absolute value is smaller. It could be due to the smaller carrier density, or the simplified model for the electronic scattering with impurities that we have considered. Fig. 3 shows ρ_{xx} versus B for different MW frequencies. The upper figure corresponds to a range of small values and the lower figure to large ones. In both cases ZRS are reproduced very clearly for $j = 1$.

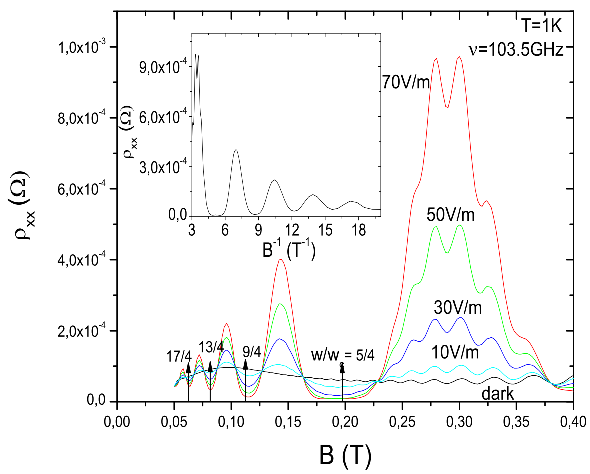

FIG. 2: Calculated magnetoresistivity ρ_{xx} as a function of B, for different MW intensities but for the same frequency $\nu = 103.5GHz$. The darkness case is also presented. In the inset we show ρ_{xx} vs B^{-1}, which is roughly periodic in B^{-1} (T=1K).

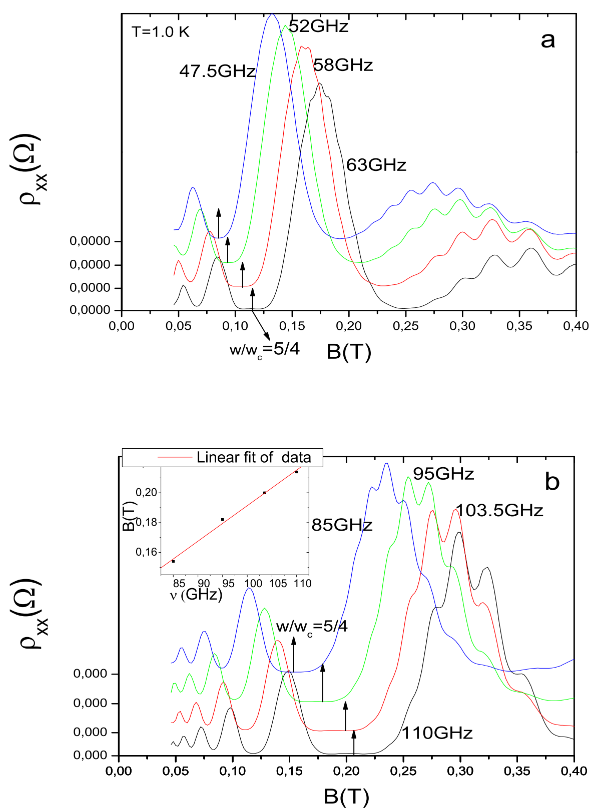

FIG. 3: Magnetoresistivity response for different MW frequencies. a) corresponds to a range of low w values and b) to higher ones. In both cases, ZRS are reproduced (T=1K).

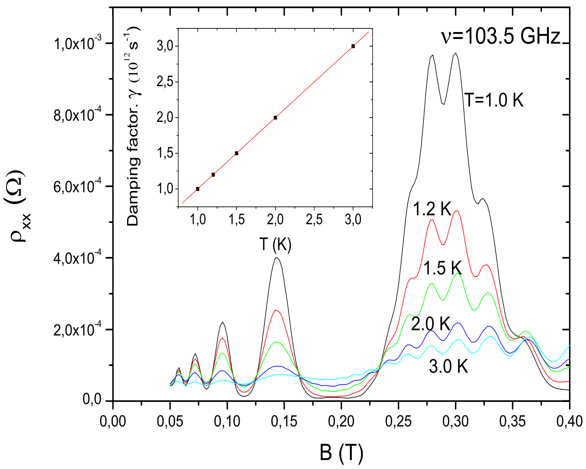

FIG. 4: ρ_{xx} versus B for different temperatures with constant power excitation. The oscillations get smaller as T is increased, but the positions of the minima stay constant. In the inset we show a calculated linear response between γ and T.

Minima positions shift as w is altered, maintaining the ratio $\frac{w}{w_c} = j + \frac{1}{4}$. This is shown in the inset, where the magnetic field B corresponding to ZRS (j=1) for four different MW frequencies is plotted as a function of w. As in experimental results presented in ref. [1], a quite reasonable linear fit is achieved.
The dependence of ρ_{xx} on T (Fig. 4), has been obtained at $\nu = 103.5$ GHz. As T increases ρ_{xx} is softened, and eventually almost disappears. The explanation can be readily obtained through the damping parameter γ. When the electronic orbits are oscillating harmonically due to the time-dependent external force, interaction with acoustic phonons occurs. This interaction acts as a damping for the orbits' movement. As T increases, the lattice-orbit interaction strengthens and the damping of orbit dynamics will be stronger as well, giving a progressive reduction in the MW-induced ρ_{xx} response. We have considered a linear dependence between γ and T as in the experiments by Studenikin et al[3].
Using our model, it is now possible to shed some light on the peculiar dependence of ρ_{xx} on B. According to our calculations we have found an approximately linear relation between ρ_{xx} maxima and B, and we can therefore express the corresponding dependence as: $\rho_{xx} \propto B\cos(w\tau) \propto B\cos\left(\frac{w}{B}\right)$. Looking at the cosine argument $(w\tau)$ it is clear that if we change w, the minima positions will change as well. Regarding $\tau = \frac{1}{W_{n,m}}$ we can say that the scattering transition rate $W_{n,m}$ is mostly dependent on sample and scattering variables, and that specific minima positions will be a function of those variables. In this way we expect that for significantly different samples, minima positions and other features of ρ_{xx} oscillations will change, explaining the discrepancy observed between different experiments.
In summary, we have presented a new theoretical model whose main foundations are the exact solution for the quantum harmonic oscillator in the presence of MW radiation and elastic scattering due to randomly distributed charged impurities. This model gives a description of the electronic orbit dynamics which is crucial to explain the physical origin of ZRS. We are able to reproduce most experimental results, including ρ_{xx} oscillations, minima positions and their dependence on w, MW intensity and T.
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